On Population Balance Modeling of Membrane Bioreactor Operation with Periodic Back-Washing

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Introduction

The membrane bioreactor (MBR) is a system that combines biological treatment with membrane filtration into a single process.[1] A basic type of this bioreactor consists of a tank, where the activated sludge biological process takes place, with a wastewater feed and clean water effluent stream,[2] which is obtained through a membrane module immersed in the tank. The effluent water is drawn through the membrane by the application of mild suction. In addition, air in the form of bubbles is fed in the liquid to provide oxygen (electron acceptor) for the bioreactions and to facilitate the circulation of the mixed liquor. Major advantages of the membrane bioreactor system over the conventional activated sludge process include the independent control of residence times of sludge particles and pure liquid. However, during membrane bioreactor operation, solids tend to be accumulated on the membrane causing clean water flux reduction. For this reason, the common operating mode involves a periodic back-washing, aiming at breaking the deposited layer (into ‘‘particles’’), to avoid excessive flux reduction, thus, effectively controlling the membrane bioreactor operation. In parallel, the inherent process limitation, related to the diffusive transport of the reactants into the sludge particles,[3,4] is expected to be affected by the particle-size distribution, which is considered an important parameter influencing the bioreaction efficiency. In this connection, air bubbling is beneficial as it tends to increase the turbulence level in the tank, in addition to the aforementioned benefits. However, despite the back-washing, an irreversible biofilm tends to slowly buildup on the membranes leading to a long-term reduction of the bioreactor efficiency. Therefore, considerable effort has been devoted in the literature to the investigation and modeling of the biofilm development.

A fruitful approach to modeling the aforementioned basic aspects of the complicated MBR system is through population balances, which have been used to advantage in many other areas of chemical engineering. Population balances in modeling of activated sludge have been previously used only in a few publications, notably those for studying the size distribution of sludge particles with focus on the determination of the coagulation kernel in conventional activated sludge processes.[5,6] Therefore, the scope of this work, based on a population balance formulation, is to present a qualitative model for the operation of the periodically back-washed membrane bioreactor, as a first step for further development. The biofilm growth is ignored, that is, assuming that it occurs over a much longer time scale than those considered here or that 100% cleaning efficiency of backwashing is achieved. Thus, the operation of membrane bioreactor is simplified to focus on the effect of intraparticle transport limitation on the bioreactor efficiency. In particular, the

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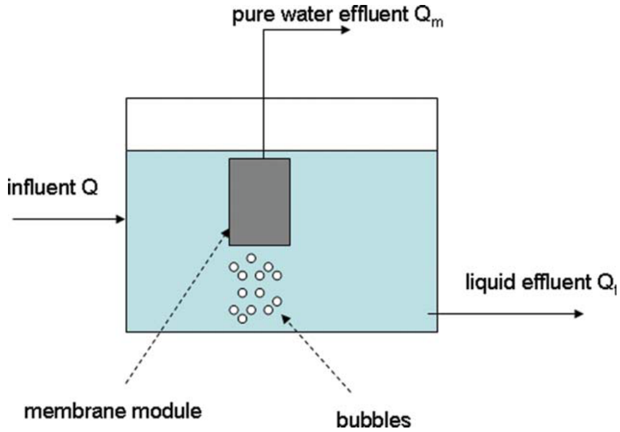

Figure 1. A schematic diagram of the membrane bioreactor.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

modeling approach is aimed at improving our understanding of the interrelation between the key operating parameters and at identifying important features of the process, for further study.

Problem Formulation

A simple schematic diagram of the membrane bioreactor under consideration is shown in Figure 1. A more detailed description of a typical MBR setup can be found elsewhere.[7] The volumetric inlet flow rate is denoted as Q, and the volumetric outlet flow rates through the membrane module and through the tank outflow are designated by Q_m and Q_t, respectively. The volume of the liquid in the tank is V, and the particle-size distribution (PSD) of the suspended biomass is denoted as $f(x,t)$. This is a number density function expressing the number concentration of particles, with volumes between x and $x + dx$, as $f(x,t)dx$. In the following, the mass fraction of the suspended biomass will be considered small enough to permit neglecting its influence on the liquid density; this is realistic for the typical range of sludge concentration 2 to 20 g/L in MBR.[2] In addition, for the purposes of this work, complete mixing conditions in the reactor are considered. A more detailed fluid mechanical modeling approach would employ compartmental modeling[8] to account for the nonuniform flow conditions in the reactor.

The operation of the membrane bioreactor exhibits an externally imposed back-flow periodicity with period T. During the first part t_1 of this period (i.e., filtration) a cake layer grows on the membrane fibers surface (only by deposition since the time scale is relatively small for biochemical growth). The filtration flow rate Q_{m1} during t_1 is assumed to be constant. This assumption is exact for constant flux reactor operation. In case of constant membrane pressure drop operation, the increase of the cake layer during t_1 leads to a reduction of the filtration flow rate, but this variation of Q_{m1} may be ignored since the fractional change of the flow resistance during t_1 is very small. During the second part of period T, which is of duration $t_2 = T - t_1$, a relatively higher pressure is applied at the permeate side leading to a reverse flow through the membrane (i.e., back-pulsing). The flow rate during the back-pulsing is designated as Q_{m2}. It is also assumed that during back-pulsing the biofilm is completely disintegrated, and its material is dispersed to the bulk of the bioreactor.

From the earlier description, it is evident that the mass balances for the tank are inherently transient due to the transient nature of Q_m, which mathematically can be expressed as

$$Q_m = Q_{m1}U(0 \le t - iT \le t_1) - Q_{m2}(1 - U(0 \le t - iT \le t_1)) \tag{1}$$

where U is a generalized function (like a Heavyside function), taking the values 1 and 0 when its argument is true or false, respectively, and i is the integer part of the quotient t/T (i.e., the expression $t - iT$ denotes the quotient t/T). The time t is

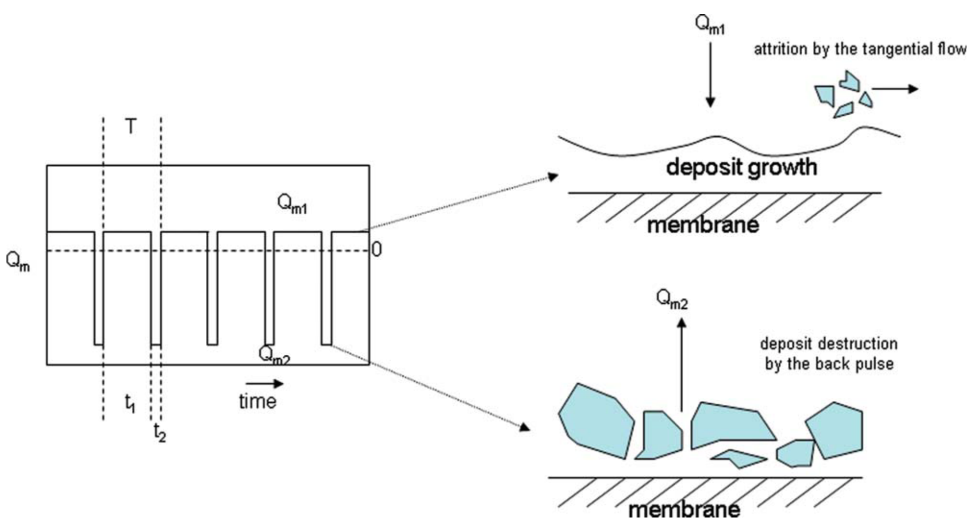

Figure 2. A simplified pictorial representation of phenomena occurring at the membrane surface.

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measured from the end of a back-pulsing cycle. Although the backwashing time t_2 does not explicitly appear in Eq. 1 (it appears implicitly through the period $T = t_1 + t_2$), a close examination of this equation reveals that it describes a periodic function Q_m with period T and values Q_{m1} for the first part of the period, with duration t_1 and $-Q_{m2}$ for the second part of the period with duration t_2, as depicted in the profile of Figure 2.

Considering the preceding discussion on MBR operation, the mass balances of liquid and suspended biomass in the reactor can be written as follows.

Liquid mass balance

$$\frac{dV}{dt} = Q - Q_t - Q_{m1}U(0 \le t - iT \le t_1) + Q_{m2}(1 - U(0 \le t - iT \le t_1)) \quad (2)$$

Sludge mass balance

$$\frac{dVf}{dt} = -Q_t f - (Q_{m1}f - A(x,t))U(0 \le t - iT \le t_1) + Q_{m2}E(x,t)(1 - U(0 \le t - iT \le t_1)) + VS(x,t) \quad (3)$$

Here $A(x,t)$ denotes the rate of sludge fragments of volume x entering the bulk of the reactor due to the "erosion" of the deposit layer by the shear flow during the deposit growth period. The function $E(x,t)$ denotes the concentration of the fragments of volume x (resulting from the disintegration of the film) in the back-washing stream. Both functions $A(x,t)$ and $E(x,t)$ are periodic with period T; the former is non zero for the first part (t_1) of the period, and the latter for the second part (t_2). Finally, the function $S(x,t)$ is a generalized volumetric source (or sink) term of sludge particles. The equation for the evolution of the cake layer volume V_B follows, where no biochemical reactions in this are considered due to their slow kinetics compared with the back-washing frequency

$$\frac{dV_B}{dt} = \left[Q_{m1}\int_0^\infty xf(x,t)dx - \int_0^\infty xA(x,t)dx\right]U(0 \le t - iT \le t_1) - Q_{m2}\int_0^\infty xE(x,t)dx(1 - U(0 \le t - iT \le t_1)) \quad (4)$$

Figure 2 depicts the phenomena occurring on the membrane fiber surfaces, albeit in a simplified manner, to better understand what is described by the aforementioned set of equations. The functions $A(x,t)$ and $E(x,t)$ must satisfy the following condition to conserve the suspended biomass

$$\int_0^{t_1}\int_0^\infty \left[Q_{m1}xf(x,t) - xA(x,t)\right]dxdt = \int_{t_1}^{T}\int_0^\infty Q_{m2}xE(x,t)dxdt \quad (5)$$

In principle, the aforementioned system of ordinary differential equations can be integrated numerically in time to determine the evolution of variables of interest (the concentration and the particle-size distribution of the suspended biomass). However, this direct approach for the solution of the problem is quite ineffective since four quite different and distinct time scales are involved, to which due attention should be given. The shortest time scale t_2 is the time of back-washing, the second time scale is the period T, the third time scale is the residence time with respect to the outflow through the membrane (V/Q_{m1}), and the longest time scale is the residence time with respect to the tank outflow (V/Q_t). These time scales extend from a few seconds to several days, but they have the exploitable advantage that $t_2 \ll T \ll V/Q_{m1} \ll V/Q_t$.

The first step, to simplify the problem, is to consider the case in the limit $t_2/T \to 0$. In this limit, the flow rate during the back-washing can be ignored; that is, $Q_m = Q_{m1}$, $Q_{m2} = 0$. Thus, the back-washing is assumed to be instantaneous leading to $Q_{m2}E(x,t) = E(x)\delta(t - T)$. Here $E(x)$ is the total number of fragments of volume x produced during each back-washing event.

The second step is to follow a procedure typical for problems with multiple time scales (from turbulence[9] to nonlinear pendulum[10]); i.e., "the coarse graining of time". We consider a new time scale t (the same symbol is used for clarity) with its finest structure including several periods T. What we consider in the new problem as "instantaneous" is actually the average in at least one period T. In the new time scale, the biofilm dynamics can be ignored completely (since it occurs at times smaller than T), and its existence can be replaced by a source of fragments in the bulk flow. The new set of equations is derived by taking the integral of Eqs. 2–4 for one period T; that is

$$\frac{dV}{dt} = Q - Q_m - Q_t \quad (6)$$

$$\frac{dVf}{dt} = -Q_m f - Q_t f + Q_m F(x) + VS(x,t) \quad (7)$$

where $F(x) = [E(x) + \int_0^T A(x,t)dt]/(TQ_m)$. This function must conform to the following simple mass conservation requirement

$$\int_0^\infty xF(x,t)dx = \int_0^\infty xf(x,t)dx \quad (8)$$

In the macroscopic time scale, the deposit operates as a redistributor of the size of the sludge particles, receiving particles with distribution $f(x)$, and returning fragments with distribution $F(x)$. The sludge particles in the bulk of the reactor undergo simultaneously coagulation and breakage due to the turbulent flow field in the tank. The turbulence in the tank is induced by the macroscopic flow and by the injected bubbles. The literature on coagulation and breakage of flocs in a turbulent flow field is very extensive; thus, the following expression for the source term corresponding to coagulation and breakage can be employed[11]

Coagulation

$$S_c = 0.31\alpha\left(\frac{\varepsilon}{v}\right)^{1/2} \times \left[\frac{1}{2}\int_0^x (y^{1/3} + (x-y)^{1/3})^3 f(y,t) f(x-y,t) dy - f(x,t) \times \int_0^\infty (x^{1/3} + y^{1/3})^3 f(y,t) dy\right] \quad (9)$$

Breakage

$$S_b = c\left(\frac{\varepsilon}{v}\right)^{n/2} 2\int_x^\infty y^{-2/3} f(y,t) dy - x^{1/3} f(x,t) \quad (10)$$

where v is the kinematic viscosity of the liquid. The first term of Eq. 9 refers to the new particles appearing after a coagulation event, and the second term to the lost particles during this event. Similarly, the first and second terms of Eq. 11 refer to appearing and disappearing particles during a breakage event. The coagulation and breakage frequencies are obtained from the relevant literature.[11] It is noted that, due to the well-mixing hypothesis, the average value of turbulent dissipation rate ε is used in the aforementioned equation. However, it is well known that significant spatial nonuniformity of ε requires more advanced mixing models in the tank.[8] The parameter α is the collision efficiency, which for dense particles can be much smaller than unity due to viscous interactions, but for porous particles it is close to unity. The hydraulic permeability of the suspended biomass particles must be known to obtain a better estimate for α. The breakage model includes more empirical parameters. The exponent n takes values from 1 to 2 depending on the type of particles (i.e., it is a function of particle strength,[11] and it can be determined experimentally. It is also assumed that the breakage rate is proportional to the particle radius and that two fragments result per fragmentation event. Additionally, all fragment sizes are considered to have the same probability. The aforementioned assumptions are typical for turbulent breakage of flocs, and they can be easily relaxed in case additional information is obtained for the breakage functions, for the problem at hand. The source term $S = S_c + S_b$ must be added to Eq. 7.

At this point the model for the tank dynamics with ''inactivated'' suspended biomass has been completed. If the operation of the reactor in the case of a simple bioreaction is to be modeled, a growth operator of the suspended biomass particles (due to biomass production) must be added to the source term S. In addition, mass balances for the carbon source and the electron acceptor are needed to complete the problem description. For such a reaction, a steady state is established in which the biomass particle production is counterbalanced by the removal of sludge with the tank outflow stream. The final mathematical formulation has the form of the growth-coagulation-breakage equation in a continuous stirred tank which is usually encountered in the crystallization literature.[12] This complete problem formulation will not be attempted in this communication. The simpler case of a reaction rate very small (so that no appreciable increase of the total biomass occurs in the reactor) will be considered here to show the effect of the tank operating parameters on the particle-size distribution and correspondingly on the bioreactor efficiency. In the particular case of zero biomass production (negligibly small with respect to the existing biomass in the system) a steady state can be considered even without the tank outflow stream, which may be totally ignored since the residence time V/Q_t is larger than the other time scales of the system. In this case $Q = Q_m$ and the governing steady state equation for the particle-size distribution is as follows; i.e., particles entering the dispersion from disintegration of the deposit, minus particles leaving the dispersion to be deposited on the membrane, equal to particles produced by bulk coagulation plus particles produced by bulk breakage

$$Q(F - f) + VS_c + VS_b = 0 \quad (11)$$

This is the steady state coagulation-breakage equation in a continuous stirred-tank reactor (CSTR) that has been studied mainly in the context of liquid-liquid dispersions.[13]

Under conditions of excess of electron acceptor, and a very small concentration of the carbon source, the kinetics of bioreaction can be assumed to be of first-order. The diffusion coefficient D of the carbon source is typically less than 10^{-9} m^2/s, and the typical bioreaction rate constant k is[14] of the order of 10^{-4} s^{-1}, which implies that diffusion limitations start appearing even for particles with radius as small as a few hundred microns. The actual bioreaction rate results by multiplying the nominal rate with an effectiveness factor. It is noted that the bioreaction rate is finite, and only its capability to increase appreciably the biomass concentration (particle volume) is considered small. This effectiveness factor is computed from the solution of the diffusion-reaction equation for the suspended biomass particle; for the first-order reaction, a well known form is obtained[15]

$$e_f = Z(R\sqrt{k/D}/3) \quad (12)$$

where R is the radius of the biomass particle and

$$Z(y) = \frac{1}{y}\left(\frac{1}{tanh(3y)} - \frac{1}{3y}\right) \quad (13)$$

The aforementioned expressions correspond to the well known[15] effectiveness factor for a first-order reaction in a spherical catalyst particle. The average efficiency of the bioreaction in the tank can be computed, by weighting the effectiveness factor over the particle-size distribution, from the relation

$$e_{f,ave} = \int_0^\infty x f(x) Z((3x/4\pi)^{1/3}\sqrt{k/D}) dx/M \quad (14)$$

where

$$M = \int_0^\infty x f(x,t) dx$$

According to Eq. 11 there is a constant mass of particles in the tank, but the particle-size distribution depends on three competing phenomena. Thus, if the extent of bulk coagulation and breakage is small, the steady state PSD is dominated by the deposit fragment distribution. In the other limit, the PSD is determined by the coagulation-breakage equilibrium in the bulk. Eq. 11 must be solved to estimate the steady state PSD.

To continue, the following nondimensionalization is introduced. By definition, N is the number concentration of the deposit fragment-size distribution F ($N = \int_0^\infty F(x)dx$), M is the volume fraction of these fragments in the liquid, and $x_o = M/N$ the average deposit fragment volume. It will be noted that using Eq. 11 one can show that $\int_0^\infty xF(x)dx = M$, since coagulation and breakage are volume- conserving phenomena. Using the following dimensionless variables

$$\bar{x} = x/x_o, \quad \bar{f}(\bar{x}) = \frac{x_o f(x)}{N}, \quad \bar{F}(\bar{x}) = \frac{x_o F(x)}{N}$$

Eq. 11 takes the form

$$-\bar{f}(\bar{x}) + \bar{F}(\bar{x}) + Z_1 \left[\frac{1}{2} \int_0^{\bar{x}} (\bar{y}^{1/3} + (\bar{x}-\bar{y})^{1/3})^3 \times \bar{f}(\bar{y})\bar{f}(\bar{x}-\bar{y})d\bar{y} - \bar{f}(\bar{x}) \int_0^\infty (\bar{x}^{1/3} + \bar{y}^{1/3})^3 f(\bar{y})d\bar{y} \right] + Z_2 2 \int_{\bar{x}}^\infty \bar{y}^{-2/3}\bar{f}(\bar{y})d\bar{y} - \bar{x}^{1/3}\bar{f}(\bar{x}) = 0 \quad (15)$$

where

$$Z_1 = \frac{0.31\alpha VM}{Q}\left(\frac{\varepsilon}{v}\right)^{1/2}$$

and

$$Z_2 = \frac{cVx_o^{1/3}}{Q}\left(\frac{\varepsilon}{v}\right)^{n/2}$$

The parameters Z_1 and Z_2 are the dimensionless extent of coagulation and of bulk fragmentation, respectively. Eq. 15 must be solved numerically for the suspended biomass particle-size distribution $\bar{f}(\bar{x})$. Typically, computationally demanding techniques, such as sectional methods or methods of finite elements, are used for this purpose.[16] Here the method of moments[17] will be used due to its computational efficiency and its compatibility with the qualitative nature of this work. The following log-normal shape is assumed for the deposit fragments distribution

$$\bar{F}(\bar{x}) = \frac{1}{\sqrt{2\pi\sigma_o}}\frac{1}{\bar{x}}exp\left[-\frac{1}{2\sigma_o}(ln(\bar{x}) + \frac{\sigma_o}{2})^2\right] \quad (16)$$

This type of fragments distribution is consistent with what is known for fragmentation of brittle objects.[18] In case more information becomes available from operating MBR systems, suggesting a different form for the fragments distribution, its application to the proposed framework is rather straightforward. By construction, the above distribution has the zero and the first moment equal to 1. The dispersivity σ_o defined as the logarithm of its second moment is an index of the spread of the fragment distribution. For the particles in the bulk, the assumed size distribution is

$$\bar{f}(\bar{x}) = \frac{M_o}{\sqrt{2\pi\sigma}}\frac{1}{\bar{x}}exp\left[-\frac{1}{2\sigma}ln^2(\bar{x}\, M_o e^{\sigma/2})\right] \quad (17)$$

The first moment of this distribution is also 1, but the dimensionless zero moment M_o and the dispersivity σ are unknown and must be determined based on the requirement that Eq. 15 should be satisfied as best as possible. To achieve this, the integral of Eq. 15 for x from 0 to ∞ is taken, leading after some algebra, to

$$-M_o + 1 - Z_1(1 + 3M_{2/3}M_{1/3}) + Z_2 M_{1/3} = 0 \quad (18)$$

Repetition of the same procedure, after a multiplication by x^2 leads to

$$-M_2 + e^{\sigma_o} + Z_1(M_2 + 3M_{5/3}M_{4/3}) - Z_2\frac{1}{3}M_{7/3} = 0 \quad (19)$$

where

$$M_i = \int_0^\infty \bar{x}^i \bar{f}(\bar{x})d\bar{x}$$

Based on the distribution form given by Eq. 17, the moments M_i are related to M_o and σ through the equation

$$M_i = M_o^{1-i} e^{\frac{i^2-i}{2}\sigma} \quad (20)$$

Substituting this equation for i = 1/3, 2/3, 4/3, 5/3, 2, 7/3 into Eqs. 18 and 19, a system of two equations with two unknowns (M_o, σ) is obtained. For a particular set of parameters Z_1, Z_2 and σ_o, the aforementioned system is solved using the Newton-Raphson technique. After determining M_o and σ, the distribution $\bar{f}(\bar{x})$ is known from Eq. 17, and the overall effectiveness factor can be computed as

$$e_{f,ave} = \int_0^\infty \bar{x}\bar{f}(\bar{x})Z(\varphi_o \bar{x}^{1/3})d\bar{x} \quad (21)$$

where the reference Thiele modulus φ_o is defined as $\varphi_o = \frac{1}{3}(\frac{3x_o}{4\pi})^{1/3}\sqrt{\frac{k}{D}}$.

Results and Discussion

Diffusion limitations may appear even for particles of size a few hundred microns, which is typical for suspended biomass particles. In the following, the deposit fragment size will be considered fixed (corresponding to $\varphi_o = 2$), and the effect of operating parameters (i.e., the residence time $\tau = V/Q$, the suspended biomass volume fraction M, and the average turbulent energy dissipation rate ε in the tank) on the sludge particle-size distribution and the corresponding bioreaction efficiency will be presented. The value of the

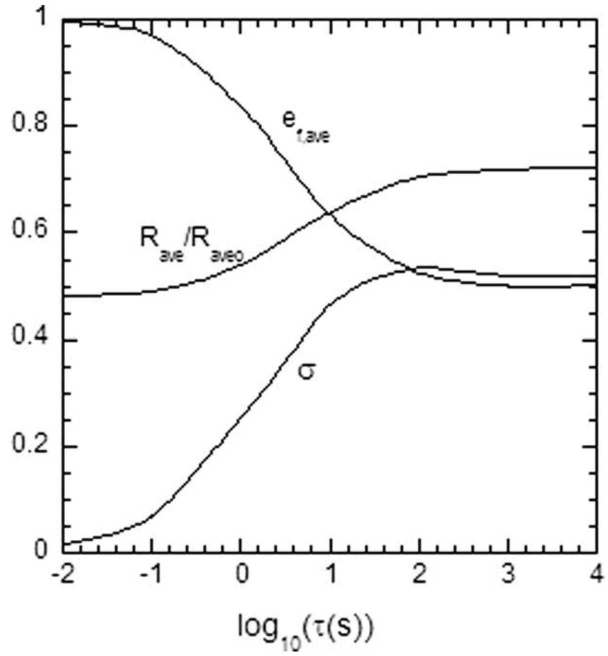

Figure 3. Influence of the reactor residence time τ on reactor characteristics; base case (M = 0.001, $\varepsilon = 0.001\ m^2/s^3$), $\sigma_o = 0$.

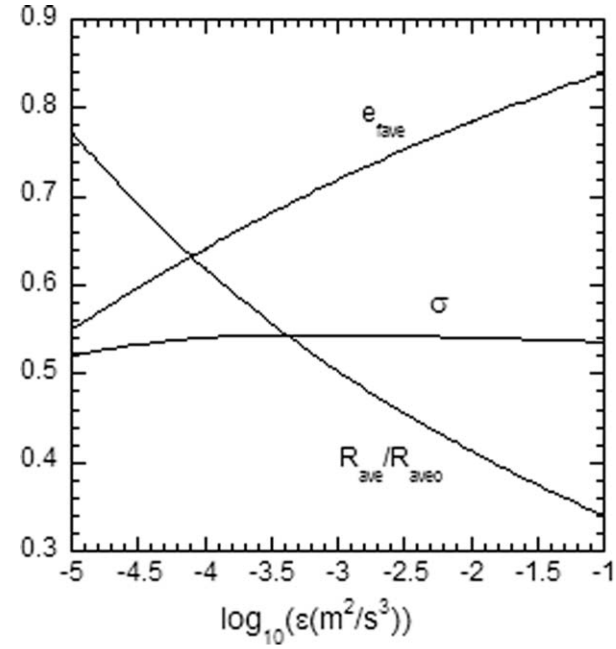

Figure 5. Influence of the energy dissipation rate ε in the tank on reactor characteristics; base case (τ = 1000 s, M = 0.001), $\sigma_o = 1$.

exponent n is taken as 2. As the *base case*, it is assumed that τ = 1000 s, M = 0.001, and $\varepsilon = 0.001\ m^2/s^3$, $cx_o^{1/3} = 0.001$. The deposit fragment distribution can be either narrow (approximated as monodisperse with $\sigma_o = 0$) or broad with $\sigma_o = 1$, which can include the effect of film erosion. The quantities computed are the dispersivity of the suspended biomass particle-size distribution σ, the ratio of the average volume equivalent radius of the suspended biomass particles to the corresponding radius of the deposit fragments R_{ave}/R_{aveo}, and the total reaction efficiency $e_{f,ave}$.

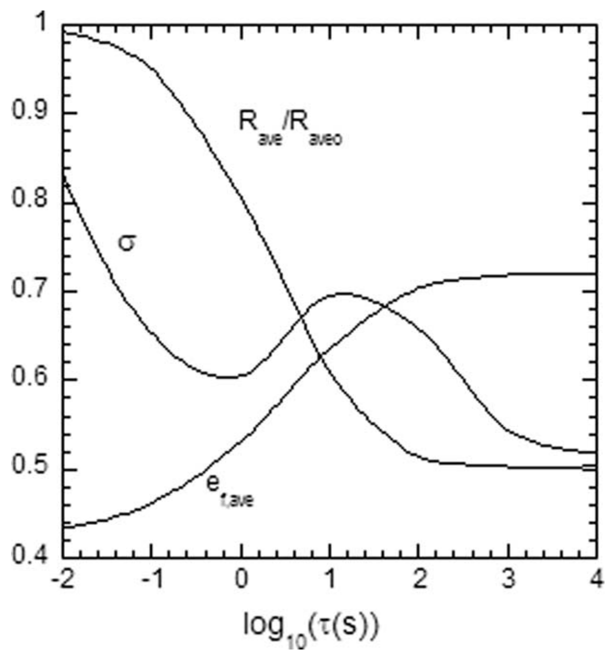

Figure 4. Influence of the reactor residence time τ on reactor characteristics; base case (M = 0.001, $\varepsilon = 0.001\ m^2/s^3$), $\sigma_o = 1$.

The effect of residence time τ on the aforementioned quantities for the base case and $\sigma_o = 0$ is shown in Figure 3. The bulk particle-size distribution features extend from the deposit fragments to those corresponding to coagulation-fragmentation equilibrium with increasing residence time. In this case, the transition point is at $\tau \sim 100$ s. At this residence time, the particles in the bulk completely "ignore" (being totally unaffected by) the fragmentation occurring on the deposit, and no change of their distribution is observed for larger residence times. The reaction efficiency increases by approximately 40% with increasing residence time. This means that breakage is crucial for increasing the efficiency of the process. In the absence of coagulation the reaction efficiency tends to one; however, coagulation is unavoidable and tends to reduce the effect of breakage. It should be noted here that in the absence of breakage one would expect an increase of particle size in the tank with respect to the deposit fragment size. In this case, Eq. 11 is the coagulation equation in a stirred tank, with the coagulation kernel having a homogeneity index 1. It is well known that this equation admits no physically meaningful solutions leading to gelling behavior.[19] In practice, this gelling behavior never occurs because other phenomena such as breakage or sedimentation eliminate large particles from the system.

Figure 4 is similar to Figure 3, the only difference being the dispersivity of the deposit fragments distribution σ_o which is one. The large τ time behavior of the shape of the particle-size distribution (which is expressed by the dispersivity σ) is exactly the same as the one of Figure 3 since it is independent of the deposit fragmentation process. The complete curves for the average radius and the reaction

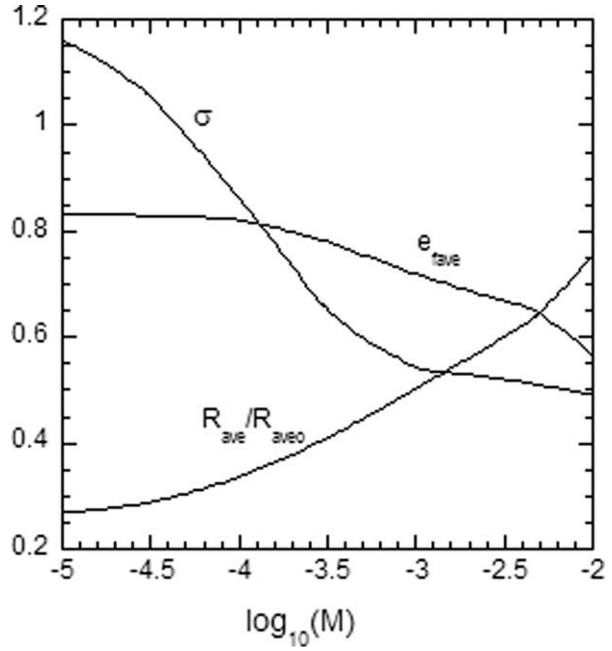

Figure 6. Influence of the suspended biomass volume fraction M on reactor characteristics; base case (τ = 1000 s, ε = 0.001 m^2/s^3), $\sigma_o = 1$.

efficiency are similar to those in Figure 3, revealing a small influence from the dispersivity σ_o. The behavior of the shape of the particle-size distribution is more complicated in this case, which is reflected in the nonmonotonic nature of the corresponding curve.

The effect of the average turbulent energy dissipation rate in the tank for the base case with $\sigma_o = 1$ is depicted in Figure 5. A broad range of ε appears to cover situations from weak bubbling to intense stirring. The residence time is long enough, so that there is no influence of the deposit fragmentation; thus, the bulk conditions reflect the dominance of coagulation-breakage equilibrium. It is noted that although coagulation increases with ε, breakage tends to increase more, leading to the reduction of particle size, and correspondingly to an increase of the reaction efficiency. It will be also pointed out, parenthetically, that the independence of the dispersivity from ε is absolutely compatible with the theory of self-similarity of the equilibrium breakage-coagulation distributions with respect to the turbulent shear rate.[20]

The effect of the volume fraction of the suspended biomass for the base case and $\sigma_o = 1$ is shown in Figure 6. The coagulation term is proportional to the suspended biomass volume fraction, whereas the breakage term does not depend on that. As a result, with increasing M the average particle size tends to increase and correspondingly the reaction efficiency tends to decrease. For small values of M the bulk coagulation disappears and the process is dominated by the bulk fragmentation and the film phenomena. The competition between these new factors leads to larger dispersivity values than those corresponding to the bulk coagulation-breakage equilibrium discussed earlier.

In conclusion, it seems that the steady state sludge particle-size distribution is insensitive to the dispersivity of the deposit fragments distribution and to the residence time in the tank (for residence times larger than a critical value). Furthermore, the increase of turbulence intensity tends to reduce the particle sizes, whereas the increase of suspended biomass particle-volume fraction tends to increase them. It is pointed out that the increase of the bioreaction efficiency with particle breakage, obtained here, refers to flocs participating in a single process. In case of flocs with different microorganisms involved in several processes, the overall behavior can be different. However, in any case the particle may be associated with transport limitations which affect the bioreactor performance. It will be also noted that the assumption of complete mixing is not crucial for the results presented here (i.e., the relation between the output variables and ε), since it only modifies the value of ε used. The conventional average turbulent energy dissipation rate is appropriate to be used in Eqs. 9 and 10, only for relatively uniform flow fields. In the general case of nonuniform flow fields, ε must be computed in a more complicated way, but the equations and the corresponding analysis are still valid.

Concluding Remarks

A mathematical model is developed for the operation of a type of membrane bioreactor in which the bioreactions occur exclusively in the suspended biomass particles. This model is based on the exploitation of the disparate time scales of the process and on the use of population balances to describe the evolution of the suspended biomass particle-size distribution, and also to incorporate the transport limitations to bioreactor modeling. The model presented herein is solved for the simplified case of negligibly small carbon source concentration, using the method of moments. Several results are obtained revealing the influence of key process operating parameters on the particle-size distribution of suspended biomass and on bioreaction efficiency. This simplified model can represent and explain qualitatively the expected behavior of the process. However, it should be reiterated that the scope of this work is not a detailed model of the membrane bioreactor operation, but the derivation of a simplified one that allows focusing on the intraparticle transport limitations and their impact on the bioreactor efficiency. The proposed approach can be considered a first step for the development of a new class of advanced models for bioreactor dynamics, by taking into account the particle-size distribution of suspended biomass and its effect on intraparticle processes.

Notation

$A(x,t)$ = rate of attrition of fragments (of volume x) from the deposited film-density function, # particles/m^3 s
c = breakage constant, s^{n-1}/m
D = diffusion coefficient of the carbon source in the particle, m^2/s
e_f = effectiveness factor
$e_{f,\mathrm{ave}}$ = average effectiveness factor in the reactor
$E(x,t)$ = number density concentration of particles of volume x in the back pulse stream, # particles/m^6
$E(x)$ = number density of fragments produced per backwashing event, # particles/m^3
$F(x)$ = normalized number concentration density function corresponding to the products of deposit attrition and fragmentation, # particles/m^6

$f(x)$ = number concentration density of particles of volume x, # particles/m^6
k = kinetic constant of the bioreaction, 1/s
M = volume fraction of the particles in the dispersion
M_i = dimensionless moments of order i of the function, f(x)
N = total particle concentration corresponding to F(x), # /m^3
Q = volumetric flow rate of the inlet stream, m^3/s
Q_{m1} = volumetric flow rate through the membrane at the filtration stage, m^3/s
Q_{m2} = volumetric flow rate through the membrane at the backwashing stage, m^3/s
Q_t = volumetric flow rate of the outlet stream, m^3/s
R_{ave} = average particle radius in the reactor, m
R_{aveo} = particle radius corresponding to x_o, m
$S(x)$ = bulk number source density of particles of volume x, # particles/m^6 s
S_b = bulk number source density of particles due to breakage, # particles/m^6 s
S_c = bulk number source density of particles due to coagulation, # particles/m^6 s
T = duration of a filtration-backwashing cycle, s
t = time, s
t_1 = duration of the filtration stage, s
t_2 = duration of the backwashing stage, s
V = volume of the dispersion in the reactor, m^3
V_B = volume of the deposited film on the membrane, m^3
x = particle volume, m^3
x_o = average particle volume corresponding to fragment distribution F(x), m^3
α = particle collision efficiency
ε = turbulent energy dissipation rate, m^2/s^3
ν = kinematic viscosity of the dispersion, m^2/s
σ = dispersivity of the particle-size distribution in the reactor
σ_o = dispersivity of the fragment distribution F(x)
τ = residence time in the reactor (= V/Q), s
ϕ = Thiele modulus
ϕ_o = reference Thiele modulus corresponding to particle volume x_o
overbar = dimensionless variables

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